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A Qualitative Analysis of Grade Nine Students Difficulties in Geometry

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Abstract: **This study aimed to determine the difficulties experienced by grade nine students in geometry. A total of 30 grade nine students participated in answering the open-ended questionnaire and Focus Group Discussion. The results of the study revealed that students like mathematics more specifically geometry. They recognize the importance of the subject. However, some students find it difficult. They find mathematics and even geometry abstract and boring. However, some students said that they like geometry because they like their geometry teacher. According to them, their geometry teacher teaches well the subject. They also like the subject because, they like geometrical figures, they like sketching and drawing of these figures. On the other hand, they don't like the subject because according to them the subject is difficult. Reasoning and proving are very difficult for them. This means that they lack logical and critical thinking and they are poor in this aspect. Furthermore, geometry is difficult for them because they cannot recall the postulates and theorems that they need to support for different reasons. Along with this line, it is recommended that geometry teachers are encouraged to continue motivating their students to pay attention and put more interest in mathematics especially geometry. Geometry teachers are also encouraged to develop learning exercises in reasoning and proving so that performance of the students on this topic will be improved.**

Keywords: **Difficulties, Geometry, Grade nine students, Qualitative analysis.**

I. INTRODUCTION

In the secondary level, geometry is an important area in the school mathematics curriculum. Throughout history, it has had great importance in people's lives, originating with the need of human beings to specify quantities, to measure figures, land and earth, and make maps.

Nowadays, geometry maintains its value with greater importance. In order to represent and solve problems in areas topics of mathematics like trigonometry and in daily life situations, sound geometry knowledge is necessary. Geometry is also used in other disciplines such as science (e.g., optics), geography (e.g., making maps), music (e.g., the pattern of the notes), art (e.g., making models), construction, architecture, gardening, and traffic signs.

Students face geometry wherever they go. Artists, builders, designers, masons, machinists, structural engineers, and writers all use geometry on the job. School geometry is the study of spatial objects, relationships, transformations that have been formalized, and the axiomatic mathematical system that have been constructed to represent them. The National Council of Teachers of Mathematics (NCTM, 2000) has emphasized the importance of geometry in school mathematics by stating, "Geometry and spatial sense are fundamental components of mathematics learning. They offer ways to interpret and reflect on our physical environment."

Geometry allows students to develop insight to understand other mathematical concepts and connect ideas across different areas of mathematics (Mammana & Villiani, 1998; Muschla & Muschla 2000; NCTM, 2000). Furthermore, many ideas like symmetry or generalization can help students increase insights into the nature and beauty of mathematics (NCTM,

2000). Even if one does not plan to become a mathematician, he or she needs to develop visualization and reasoning abilities, and appreciation of nature.

Every human being needs some geometry intuition to understand and interpret the world and our physical environment. The importance of geometry is best stated by an inscription above the door of Plato's school, "Let no one destitute of geometry enter my doors (Burton, 1999; p.79)." Although geometry is an important area and much effort is exerted in teaching geometry, evidence from numerous research studies makes it clear that many students' geometrical understanding is not at the level they need or are expected to be (Burger & Shaugnessy, 1986; Clements & Battissa, 1992; Mitchelmore, 1997; NCTM, 1989; Prescott, Mitchelmore, & White, 2002).

In the Philippines and in almost all of the schools, particularly students in the Division of Sagay City have experienced difficulties in mathematics more specifically in Geometry. Thus, a qualitative research was conducted to determine and identify aspects in geometry which are difficult for the students.

II. THE PROBLEM

The main purpose of this study was to analyze qualitatively the difficulties experienced by the third year high school students in geometry. Specifically, this study aims to determine whether or not third year high school students like mathematics more specifically geometry. The study also determined third year high students' perceptions of geometry as a subject in the curriculum. Furthermore, this study identified topics which third year high school students find it easy and difficult. Lastly, identified factors that influence third year high school students' difficulties in learning geometry were determined and analyzed.

III. THEORETICAL FRAMEWORK

This study was anchored on Van Hiele Theory of Geometric Teaching and Learning. The van Hiele theory was developed in the 1950s by two Dutch mathematics teachers, Pierre and Dina van Hiele. The theory attempts to explain how students learn geometry and why many have difficulty with higher-level cognitive processes, especially when they are expected to give geometric proofs. According to this theory, the development of the mathematical thought process, especially geometry, can be divided into five levels: level 0 (visualization), level 1 (analysis), level 2 (abstraction), level 3 (deduction), level 4 (rigor). The levels are sometimes described as running from level 1 to level 5, creating some confusion as to which level is being discussed. There are also alternative names given to each level. Origami and the Van Hiele Theory after the IOC established the Origametria program in schools, many parallels were found with the van Hiele method of teaching geometry. The following is an overview of the van Hiele levels

Level O (Basic Lad): Visualization At this initial stage, students are aware of space only as something that exists around them. Geometric concepts are viewed as total entities rather than as having components or attributes. Geometric figures, for example, are recognized by their shape as a whole, that is, by their physical appearance, not by their parts or properties. A person functioning at this level can learn geometric vocabulary, can identify specified shapes, and given a figure, can reproduce it. For example, given the diagrams student at this level would be able to recognize that there are squares in (a) and rectangles in (b) because these are similar in shape to previously encountered squares &d rectangles. Furthermore, given a geoboard or paper, the student could copy the shapes. A person at this stage, however, would not recognize that the figures have right angles or that opposite sides are parallel.

Level 1: Analysis. At level 1, an analysis of geometric concepts begins. For example, through observation and experimentation students begin to discern the characteristics of figures. These emerging properties are then used to conceptualize classes of shapes. Thus figures are recognized as having parts and are recognized by their parts. Given a grid of parallelograms, students could, by "coloring" the equal angles, "establish" that the opposite angles of parallelograms are equal. After using several such examples, students could make generalizations for the class of parallelograms. Relationships between properties, however, cannot yet be explained by students at this level, interrelationships between figures are still not seen, and definitions are not yet understood.

Level 2: Informal Deduction. At this level, students can establish the interrelationships of properties both within figures (e.g., in a quadrilateral, opposite sides being parallel necessitates opposite angles being equal) and among figures (a square is a rectangle because it has all the properties of a rectangle). Thus they can deduce properties of a figure and recognize classes of figures. Class inclusion is understood. Definitions are meaningful. Informal arguments can be

followed and given. The student at this level, however, does not comprehend the significance of deduction as a whole or the role of axioms. Empirically obtained results are often used in conjunction with deduction techniques. Formal proofs can be followed, but students do not see how the logical order could be altered nor do they see how to construct a proof starting from different or unfamiliar premises.

Level 3: Deduction At this level, the significance of deduction as a way of establishing geometric theory within an axiomatic system is understood. The interrelationship and role of undefined terms, axioms, postulates, definition, theorems, and proof is seen. A person at this level can construct, not just memorize, proofs; the possibility of developing a proof in more than one way is seen; the interaction of necessary and sufficient conditions is understood; distinctions between a statement and its converse can be made.

Level 4: At this stage the learner can work in a variety of axiomatic systems, that is, non-Euclidean geometries can be studied, and different systems can be compared. Geometry is seen in the abstract. This last level is the least developed in the original works and has received little attention from researchers. P. M. van Hiele has acknowledged that he is interested in the first three levels in particular (Alan Hoffer, personal communication, 25 February 1985). Since the majority of high school geometry courses are taught at level 3, it is not surprising that most research has also concentrated on lower levels. Perhaps as the van Hiele model is extended to other areas (it is being applied to economics and chemistry in Holland), this last level will achieve more prominence.

IV. METHODOLOGY

Research Method

This study aims to determine the difficulties experienced by third year high school students in learning geometry. Thus, a qualitative research approaches specifically case study method was used in this study. It made use of research technique such as open-ended questionnaire and focus group discussion. Qualitative research is designed to reveal a target audience's range of behavior and the perceptions that drive it with reference to specific topics or issues. It uses in-depth studies of small groups of people to guide and support the construction of hypotheses. The results of qualitative research are descriptive rather than predictive (http://www.qrca.org/?page=whatisqualresearch downloaded August 15, 2015).

Participants of the Study

The participants of the study were the randomly selected 30 third year high school students of Sagay National High School officially enrolled during the school year 2013-2014. Participants of the study were selected from the sections handled by the researcher. These participants were the subject of an open ended questionnaire and focus group discussion.

Data Collection

To gather information from the participants, the following were undertaken by the researchers:

1. Development of an open-ended questionnaire, interview guide and questions for the proposed focus group discussion.

2. Cross-examination of the open-ended questionnaire, interview guide and questions for focus group discussion by experts in the field. This is to check whether or not questions in an open-ended questionnaire, interview guide, and focus group discussion are aligned to the purpose of the study. After which open-ended questionnaire, interview guide and items for focus group discussion was prepared.

3. Selection of the participants. In selecting teachers as participants of the study, the researchers had randomly selected 10 students from the three sections of the class she is handling.

4. Conduct of an open-ended questionnaire and interview, and focus group discussion. In the conduct of the open-ended questionnaire, interview, and focus group discussion, the researchers conducted the three activities to gather data on the scheduled class of the participants.

5. The conduct of the focus group discussion. According to Gibbs (1997), focus groups are usually pretty small to allow participation from all members. One study says that the recommended number of people per group is usually six to ten but some groups go as high as fifteen. In the conduct of the focus group discussion, the researchers conducted three separate focus group discussions.

6. Data processing. Data processing includes recording data and preparing memos, establishing codes through memos, organizing data into similar categories, and summarizing the main themes for presentation, analysis, and interpretation.

Analysis

Thematic analysis was utilized in this study. According to Braum and Clarke (2006) thematic analysis is a qualitative method used for 'identifying, analyzing and reporting patterns (themes) within data. They also state that thematic analysis is a foundational method of analysis that needed to be defined and described to solidify its place in qualitative research.

In this study, inductive thematic analysis proposed by Boyatzis (1998) was utilized by the researchers. The initial stage is coding. The coding procedure starts with reducing the raw information by written outlines of each unit of text. The text units are 'chunks' of information expressed by the participants in an open-ended questionnaire, interview, and focus group discussion, which was interpreted in terms of what the participants explicitly or implicitly is saying or responding. The next stage is the process of identifying themes. In this study, themes were identified in each of the issues presented such (a) likelihood of the subject math; (b) likelihood of the subject geometry; (c) on whether geometry is easy or a difficult subject; (d) on whether or not geometry is a boring subject; and (g) on topics in geometry they find easy and difficult; and (h) factors contributory why geometry is difficult for the students.

V. RESULTS AND DISCUSSION

Do You Like Mathematics?

Almost all of the grade nine students responded that they like mathematics. The reasons why they like mathematics vary from the fact that mathematics involve word problem solving. One participant quoted that *"one reason that I like mathemat is that I can use mathematics in their everyday life"*. This means that they can use this in buying things; they can use this in their house hold activities, and many more. In other words they like mathematics because of the significance of mathematics in their daily lives. Geometry for instance, geometry plays a very important role in radiation oncology (the study and treatment of tumors) when determining safe level of radiation to be administered to spinal cords of cancer patients (WGBH Educational Foundation, 2002).

Furthermore, they said that they like mathematics because they like the teacher. One participant said *"I am interested to attend the subject because our mathematics teachers knows how to teach mathematics easily"*. In this regard, it can be said that mathematics teachers can be a factor why students do not perform well in the subject. Mathematics is sometimes the reason why they do not like the subject. Others responded also that they like mathematics however; they find mathematics more difficult than any other subjects.

On the other hand, it cannot be denied that some of the participants boldly say that they don't like mathematics and even hate the subject. They don't like the subject because they believe that mathematics is really difficult. Their responses can be said true because national and international results of mathematics performances indicate that mathematics was among the lowest mean percentage scores in some of the areas in mathematics. As cited by one participants during the Focus Group Discussion *"I find mathematics difficult because its a little bit abstract, I could not realize the significant of xs and ys in algebra, I find it hard to conceptualize the meaning"*.

Data from the National Education Testing Center (NETRC, 2000) showed that the National Elementary Achievement Test (NEAT) results of January 2000 were disappointing. Based on the national result, English, mathematics and science were among the areas which obtained a lower percentage of performance (Gonzales, 2001).

In addition to this, NETRC as a governing body in administering nationalized achievement and aptitude test also showed that whether it is achievement or aptitude that is being assessed, what comes out consistently is the very low performance of pre-college youth in mathematics and science.

A parallel finding for the National Secondary Achievement Test (NSAT), over the years showed mathematics to be the second most difficult subject to science. This low level of performance was confirmed further in the Third International Mathematics and Science Study (TIMSS), where the Philippines ranked second and third from the bottom in science and mathematics respectively, among forty one (41) countries. According to Aro (2005) this scenario clearly shows that mathematics is one of the subjects where poor performance is more apparent among others.

Do You Like Geometry?

There are participants who said yes and there are also who said that they do not like geometry. The primary reason why they like the subject is that they like their geometry teacher. According to them their geometry teacher teaches well the subject. Although there are topics which they find difficult, however, their geometry teacher does it well. There are also other reasons why they like geometry. They like the subject because, they like geometrical figures, they like sketching and drawing of these figures.

On the other hand, they don't like the subject because according to them the subject is difficult. The very reason why they dislike the subject it's because of reasoning and proving which means that they lack logical and critical thinking. They also added that reasoning and proving are difficult because they cannot recall the postulates and theorems that they need to support for the different reasons.

According to Battista (2007) the teaching of geometry provides not only a key vehicle for developing learners' spatial thinking and visualization skills in mathematics, but also a major opportunity to develop their capability with deductive reasoning and proving. Even so, it remains uncertain what classroom factors might trigger productive mathematical reasoning and proof in school geometry lessons.

The role that geometry teaching can play in developing students' ideas about proof and proving can be illustrated by the learning progression in general use in lower secondary schools in Japan:

In Grade 7, students (aged 12-13) study selected properties of plane and solid figures informally, but logically, to establish the basis of the learning of proof.

In Grade 8, students (aged 13-14) are introduced to formal proof through studying properties of angles, parallel lines, and congruent triangles, during which they learn the structure of proofs, how to construct proofs, and how to explore and prove properties of triangles and quadrilaterals.

In Grade 9, students (aged 14-15) study similar figures, properties of circles, and the Pythagorean theorem, drawing on their consolidated capacity to use proof in geometry.

In the case of Japan, this learning progression applies to *all* students, not just the more capable students. As such, while many students in Japan are successful in learning about proof, many have difficulties. The 2008 Japanese national survey, for example, reported that about 44% of Japanese Grade 9 students could analytically/synthetically plan and construct proofs (NIER, 2008); this means that over half of students could not.

In relation to this, Jones et. al. (2014) reported that while most Grade 9 students (aged 14-15) can write down a proof, around 70% do not understand why proofs are needed (see Kunimune, 1987; Kunimune, Fujita & Jones, 2009; 2010). In this research, we capture students' understanding of proof in terms of two components: these are 'Construction of proof' and 'Generality of proof'. The first of these, 'Construction of proof in geometry', recognizes that, on the one hand, students have to learn how to 'construct' deductive arguments in geometry by knowing sufficient about definitions, assumptions, proofs, theorems, logical circularity, and so on. The second of these two components, 'Generality of proof', recognizes that, on the other hand, students also have to understand the generality of proof in geometry, including the universality and generality of geometrical theorems (proved statements), the roles of figures, the difference between formal proof and experimental verification, and so on.

Do You Believe that Geometry is a Boring Subject?

Majority of the participants find geometry boring. The very reason why they find geometry boring is that they do not understand the subject. They are not participative during the discussion because they do not understand the subject matter most especially when they are required to reason out and to make a proof. According to them their class in geometry is very quite because they are not motivated to participate because giving reasons or justifications are very difficult for them.

Another reason why Geometry is boring for them because almost of the time they copy what was written on the board because according to them they do not have books as reference materials. Only their mathematics teacher has book for their geometry class. This means that instructional materials like books are limited in the classroom.

According to Nwike and Catherine (2013) effective teaching of any subject will not only stimulates student's interest in the subject but also enhance their achievement in the examination. To achieve effective teaching and learning process, there is the need for use of instructional materials. Instructional materials are the different teaching aids or apparatus which a classroom teacher employs to facilitate his or her teaching for the achievement of the stated objective. Agun (1992) defined instructional materials as those materials which are helpful to the teachers and students and which maximize learning in various areas. The use of instructional materials in teaching is very important because it provides a concrete basis for conceptual thinking motivates people to learn and captures pupils' imagination if used correctly (Ajalla, 1997). The cognitive domain of learning involves knowledge and the development of intellectual skills. It includes the recognition of specific facts, procedural patterns and concepts that serve in the development of abilities and skills Achievement test is a test given at the end to find out the extent to which a student has achieved something, acquired certain information or mastered certain skill as a result of planned instruction or training. The importance of instructional materials in teaching and learning especially at the primary and Junior Secondary Schools cannot be over emphasized considering its effect on the development of intellectual skills on the students.

Is Geometry is Easy or Difficult?

Responses of the participants to this question reflect that they find geometry difficult. Again and again their difficulties were still on proving. They find it difficult to arrange the statements in proving logically and provide reasons for the statements. May be they find this difficult because this activity requires critical thinking and thorough analysis. Furthermore, their responses speaks that Geometry requires them to remember and possibly memorize theorems and postulates which they will use for proving.

What Topics in Geometry Do You Find Easy?

According to the participants of the study, measurements are the topics which they find easy. It is quite easy for them to determine areas, volume, perimeter, and circumference. They also find it easy to deal with geometrical figures such as triangle, square, parallelogram, trapezoid, and others. They find topics on these easy because these require low level thinking skills. These require them to recall facts and memorization of the properties of these geometrical figures. Likewise, finding areas, volumes, and perimeters are easy because these require formulas and the only process involve is purely substitution.

What Topics in Geometry Do You Find Difficult?

Although they mentioned other topics such as angles, parallel lines, perpendicular lines, and congruence. However, their responses show that the most difficult topics according to them is proving which requires them logical analysis and reasoning.

What Do You Think are the Factors that Contribute why Students Find Geometry Easy or Difficult

According to them not paying attention was one factor why students find Geometry more difficult than any other disciplines in mathematics. Their responses call for the attention of students with regard to their attitude towards mathematics in general and more specifically in Geometry. According to Nkwe (1985) there is some indication that student who are positively inclined towards a subject tend to do well in that subject. However, it cannot be concluded that positive attitude will always affect good performance. For example, the study by Kiely (1990) showed that on average a small number of pupils who were not good enough in mathematics obtained high scores in the attitude test. Some of the causes of problems encountered by pupils/students in mathematics can be attributed to the teaching methods employed by teachers and teachers themselves. In the sense that there is a tendency among teachers especially novice teachers to teach mathematics the way it was taught to them during their days as pupils. In some cases, such a problem is caused by the fact that when such teachers join their professions, they come determined and enthusiastic. To the extent that they are eager to implement the theory and skills they have acquired.

They also cited that books are another contributing factor why they find Geometry difficult. In their Geometry class their mathematics teachers was the only one who have books in Geometry. They rely to what has been copied on the board for their study materials. The fact that the can use their books during their assignments is far from the reality. Their responses in this regard specify that instructional materials like books are very important in the mathematics class. These aid them to study and work with their homework and any other related activities.

Along this line, Gonzales (2005) in his article "Getting Out of Our Dilemma" pointed out that in teaching and in learning, teachers can cut down on the teaching chores and create more opportunities for self-learning so that children can learn faster, better, more efficiently, more enjoyable, by themselves and make better use of their time. In this way, they will need less guidance from the teacher, learn for themselves and learn better and faster. A key to this rapid learning of course is proper language instruction from the beginning on the language of math or numbers and natural language and symbols in general. Once the key to these symbols is unlocked, learning becomes easy and becomes a matter of self-activity.

Nuñez (1998) also opined that nowadays, with the advent of computer aided instruction (CAI), computer access program, distance learning and modular instructions, it becomes increasingly apparent that students can no longer be taught all they want to learn within the limits of the classroom. Faced with this challenged and technological advancement in education, teachers cannot help but shift their concern to teaching students how to learn, how to be independent, how to be self-directed such that they will be equipped with the skills needed to deal with an ever-changing environment. Thus, learning materials or instructional materials such as books, etc. are of great importance to the learners. This helps this learn of their own and do their homework and assignments better.

Another factors cited by them are students study habits. They believe that their study habits will lessen their burden in Geometry and any other areas of mathematics. They also believe that mathematics is constant practice. Word problem solving is constant practice they added. Magno (2010) cited that it is a common notion that when students in the school setting study hard their grades would improve. Asian learners are expected by their parents to develop good study habits for the sake of learning as reflected in having favorable grades in school. Even poor students who have developed good study habits can perform well in school (On & Watkins, 1994). Study habits are "those activities necessary to organize and complete schoolwork tasks and to prepare for and take tests" (Robbins et al., 2002).

Do You Believe that the Teacher Teaching the Subject is a Factor?

Majority of them said that the teacher is a factor why student find difficulties in mathematics and in any other areas. They said that the way mathematics teacher approaches the subject matter and the way the teacher taught the subject contribute much to the learning and attitudes of the students towards the subject. Although there are responses of the students that their Geometry teachers is quite fast of the subject matter, yes it's a fact sometimes because in the public schools mathematics teachers have competencies and they are bound to finish what are stipulated in this competencies. However, according to their responses, their mathematics teacher sees to it that as much as possible they will understand the subject matter. (Discuss here the teaching competencies of the mathematics teacher).

On the other hand, there are also students who said that mathematics teacher is not a factor why students find difficulties in Geometry. They said that students are not paying attention to the discussion of their mathematics teacher. They added that there are students who are always absent from the class and some are cutting classes staying outside of the school doing computer games. In other word students in this regard have many activities outside of the classroom which motivates them not to attend classes.

Do You Believe that the Physical Environment of the Classroom is a Factor to why Students find Geometry Easy or Difficult?

Almost all of them said that the physical environment of the classroom is a factor why students find it difficult to learn Geometry. According to their class in Geometry is disturb and is influenced by the noise outside of the classroom caused by the students who passed by. They also said that their classroom is hot and stuffy that they cannot concentrate to listen to the discussion of their teacher because the classroom is hot and no amount of air to experience in the classroom. Furthermore, they said that sometimes their classroom is not clean, there are many waste materials like candy wrapper and papers scattered anywhere.

What Would You Like to Happen for Your Class in Geometry

These are the things that the students like to happen in their Geometry class. First all what they would like is that each of them should be provided with books. They believe that if they have books they can study ahead of time and can participate during the discussion. Another reason why they need to have books is that they can work with the homework and activities at home and beside during examination they can review the topics they have discussed.

The second thing that they want to happen is that test or short quiz should be given after the discussion so that their teacher would have an idea if they understand the lesson. By doing this they believe that their teacher can may intervention or remediation that would help them understand the lesson.

The third is, they want to have a conducive classroom to learn Geometry better. A classroom that is free of noise and is well ventilated so that students can concentrate listening to the discussion of the subject matter during class hour.

Lastly, they want that their teacher would teach them slowly so that they can internalize what has been discussed during their classes in Geometry. According to them slowly but surely so that everybody will learn the subject.

VI. CONCLUSIONS AND RECOMMENDATIONS

Majority of the students believed that mathematics is interesting. However, some students find the subject boring and uninteresting. Likewise, they disclosed that geometry boring. They find geometry as one of the most difficult subjects in the curriculum. They concluded that their difficulties were still on proving. Students' difficulties were more on arranging the statements in proving logically and provide reasons for the statements.

Furthermore, this study concludes that measurement is easy for them. They find it easy for them to determine areas, volume, perimeter, and circumference. Topics such as triangle, square, parallelogram, and trapezoid are easy for them.

Likewise, the study disclosed that not paying attention, availability of instructional materials such as books and facilities and equipment were among the factors influencing their difficulties in geometry.

It was also concluded in this study that problems encountered by pupils/students in mathematics can be attributed to the teaching methods employed by teachers and teachers themselves. In some cases, such a problem is caused by the fact that when such teachers join their professions, they come determined and enthusiastic. To the extent that they are eager to implement the theory and skills they have acquired.

Since there is some likeness in mathematics as expressed by the high school students, teachers in this regard are encouraged to continue motivating their students to pay attention and put more interest towards mathematics.

Teachers handling geometry are also encouraged to develop learning exercises in proving so that performance of the high school students on this topic will be improved.

Since interest towards the subject is a factor towards high school students difficulties in geometry, teachers in this regard are encouraged to developed learning activities like games and the use of technology so that the interest of the students will be enhanced.

Likewise, instructional materials like books as much as possible should be provided to the students because books are very important in the learning process of the high school students. Other facilities are also encouraged since these are factors considered to influence learning.

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